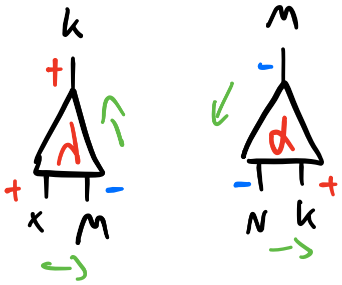
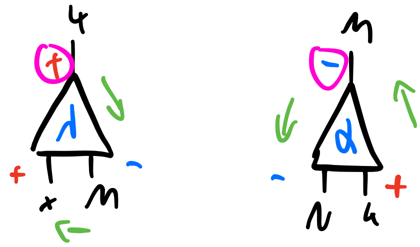
Flipping Cont



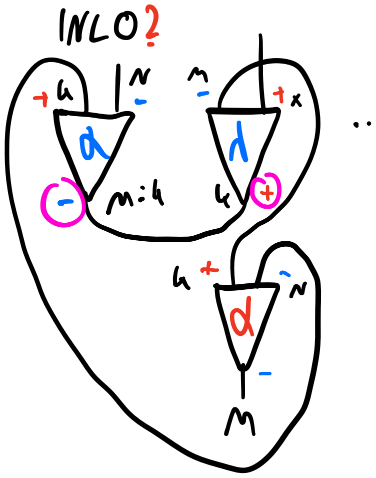
$(\mathrm{fn}\ x \Rightarrow M)_k \quad (M\ N)_k$



$(\mathrm{cofn}\ k \Rightarrow M)_x \quad (M @ k)_N$

also possible

↳ aber warum nicht $(k\ N)_M$?



$\mathrm{cofn}\ k \Rightarrow \cdot M \cdot\cdot\ ((\mathrm{INL}\ 0) @ k)$

abs vars:

$(x \Rightarrow M)_k$ × $(M\ N)_k$
$(k \Rightarrow M)_x$ $(k\ N)_M$

dual by paper

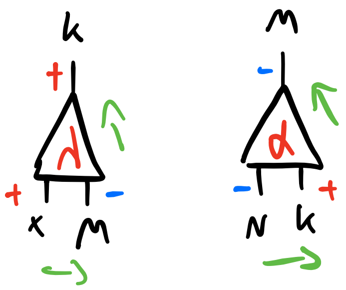
$(M \Rightarrow x)_k$ × $(k\ M)_N$
$(k \Rightarrow x)_M$ $(N\ M)_k$

$(x \Rightarrow k)_M$ × $(M\ k)_N$
$(M \Rightarrow k)_x$ $(N\ k)_M$

was ist mit cross

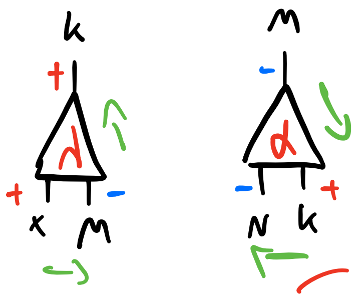
36 combos?!
↓
testen!

ist ein Kalkül sinnvoll??



$(\mathrm{fn}\ x \Rightarrow M)_k \quad (N\ k)_M$

↳ only λ ↳ only α



pp!

$(\mathrm{fn}\ x \Rightarrow M)_k \quad (M\ k)_N$

↳ only λ ↳ only α

β?

1.	$(x_+ \Rightarrow M)_{k_+}$	$(M\ N)_{k_+}$	a.
2.	$(k_+ \Rightarrow M)_{x_+}$	$(k_+\ N)_M$	b.
3.	$(M \Rightarrow x)_{k_+}$	$(k_+\ M)_N$	c.
4.	$(k_+ \Rightarrow x)_M$	$(N\ M)_{k_+}$	d.
5.	$(x_+ \Rightarrow k_+)_M$	$(M\ k_+)_N$	e.
6.	$(M \Rightarrow k_+)_{x_+}$	$(N\ k_+)_M$	f.

		k	x	k	m	m	x
		1	2	3	4	5	6
k	a	✓	✗	✓	✗	✗	✗
m	b	✗	✓	✗	✓	✓	✓
N	c	✓	✗	✓	✗	✗	✗
k	d	✓	✗	✓	✗	✗	✗
N	e	✓	✗	✓	✗	✗	✗
m	f	✗	✓	✗	✓	✓	✓

What is a β-redex? (exists in "linear" form?)

1-a

$(\lambda x.\ M)_{k_+}$

$(k\ N)_{out}$

$\Downarrow$

$((\lambda x.\ M)\ N)_{out}$

$\rightsquigarrow M[N/x]_{out}$

2-b

$(\lambda k.\ M)_x$

$(out\ N)_k$

$\Downarrow$

$(\lambda(out\ N).M)_x$

like variable that represents term after beta reduction

1-b

$(\lambda x.\ M)_k$

$(out\ N)_k$

$\Downarrow$

↯?

3-c

$(\lambda M.\ x)_k$

$(out\ k)_N$

$\Downarrow$

$(out\ (\lambda M.x))_N$

surrounding term is argument!

4d

$(\lambda k.\ x)_m$

$(N\ k)_{out}$

$\Downarrow$

↯?

3d

$(\lambda M.x)_k$

$(N\ k)_{out}$

$\Downarrow$

$(N\ (\lambda M.x))_{out}$

5e
$(\lambda x.k)_M$
$(k\ out)_N$
$\downarrow$
↯ ?

6f
$(\lambda M.k)_x$
$(N\ out)_k$
$\downarrow$
$(\lambda M.(N\ out))_x$

7f
$(\lambda x.k)_M$
$(N\ out)_k$
$\downarrow$
$(\lambda x.(N\ out))_M$

6e
$(\lambda M.k)_x$
$(k\ out)_N$
$\downarrow$
↯ ?

4c
$(\lambda k.x)_M$
$(out\ k)_N$
$\downarrow$
↯ ?

2a
$(\lambda k.M)_x$
$(k\ N)_{out}$
$\downarrow$
↯ ?

1c
$(\lambda x.M)_k$
$(out\ k)_N$
$\downarrow$
$(out\ (\lambda x.M))_N$

1d
$(\lambda x.M)_k$
$(N\ k)_{out}$
$\downarrow$
$(N\ (\lambda x.M))_{out}$

1e
$(\lambda x.M)_k$
$(k\ out)_N$
$\downarrow$
$((\lambda x.M)\ out)_N$

1f
$(\lambda x.M)_k$
$(N\ out)_k$
$\downarrow$
↯ ?

2c
$(\lambda k.M)_x$
$(out\ k)_N$
$\downarrow$
↯ ?

2d
$(\lambda k.M)_x$
$(N\ k)_{out}$
$\downarrow$
↯ ?

2e
$(\lambda k.M)_x$
$(k\ out)_N$
$\downarrow$
↯ ?

2f
$(\lambda k.M)_x$
$(N\ out)_k$
$\downarrow$
$(\lambda(N\ out).M)_x$

3a
$(\lambda M.x)_k$
$(k\ N)_{out}$
$\downarrow$
$((\lambda M.x)\ N)_{out}$

3b
$(\lambda M.x)_k$
$(out\ N)_k$
$\downarrow$
↯ ?

3e
$(\lambda M.x)_k$
$(k\ out)_N$
$\downarrow$
$((\lambda M.x)\ out)_N$

3f
$(\lambda M.x)_k$
$(N\ out)_k$
$\downarrow$
↯ ?

4a
$(\lambda k.x)_M$
$(k\ N)_{out}$
$\downarrow$
↯ ?

4b
$(\lambda k.x)_M$
$(out\ N)_k$
$\downarrow$
$(\lambda(out\ N).x)_M$

4e
$(\lambda k.x)_M$
$(k\ out)_N$
$\downarrow$
↯ ?

4f
$(\lambda k.x)_M$
$(N\ out)_k$
$\downarrow$
$(\lambda(N\ out).x)_M$

5a
$(\lambda x.k)_M$
$(k\ N)_{out}$
$\downarrow$
↯ ?

5b
$(\lambda x. k)_m$
$(\text{out } N)_k$
$\downarrow$
$(\lambda x. (\text{out } N))_m$

5c
$(\lambda x. k)_m$
$(\text{out } k)_N$
$\downarrow$
↯ ?

5d
$(\lambda x. k)_m$
$(N\ k)_{\text{out}}$
$\downarrow$
↯ ?

6a
$(\lambda m. k)_x$
$(k\ N)_{\text{out}}$
$\downarrow$
↯

6b
$(\lambda m. k)_x$
$(\text{out } N)_k$
$\downarrow$
$(\lambda m. (\text{out } N))_x$

6c
$(\lambda m. k)_x$
$(\text{out } k)_N$
$\downarrow$
↯ ?

6d
$(\lambda m. k)_x$
$(N\ k)_{\text{out}}$
$\downarrow$
↯ ?

2/4/5/6 do <u>not</u> have k => less nesting (only if app -> m)
=> 1/3 - 6/f <u>both</u> have ks in Out (abs in app)

		k	k	x	x	m	m
		1	3	2	6	4	5
k	a	✓	✓	✗	✗	✗	✗
k	d	✓	✓	✗	✗	✗	✗
N	c	✓	✓	✗	✗	✗	✗
N	e	✓	✓	✗	✗	✗	✗
m	b	✗	✗	✓	✓	✓	✓
m	f	✗	✗	✓	✓	✓	✓

• dual?

MIX 1a, 2b

1. $(x_+ \Rightarrow \underline{M})_{u_+}$ — $(\underline{M}\ \underline{N})_{u_+}$ a
2. $(k_+ \Rightarrow \underline{M})_{x_+}$ — $(k_+\ \underline{N})_{\underline{m}}$ b

β-redaction

$((\lambda x.\ M)\ N)_{out}$ $\qquad$ $(\lambda(out\ N).M)_x$

$\rightsquigarrow M[N/x]_{out}$

$\lambda k.\ M\ ((INL\ 0)\ k)$

$\downarrow$

$(\lambda k_+ .\ \underline{k_1})_{xin_+}$ 2.

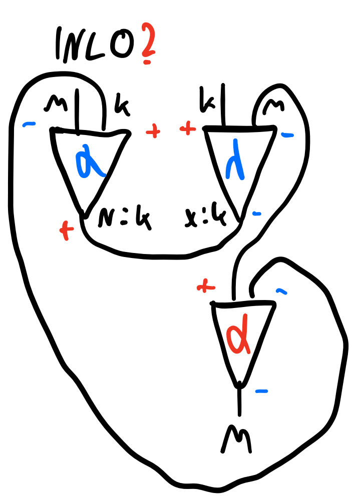
$(M_-\ k_{2-})_{k_1 +}$ a.

$(INL\ 0)_{k_3 +}$?

$(k_{3-}\ \underline{k})_{k_2 +}$

2!

cofn $k \Rightarrow$ ..M.. $((INL\ 0) @ k)$ $\rightsquigarrow$ INL 0



$\lambda x . (\lambda y . y)\, x$

<u>1-a</u>:

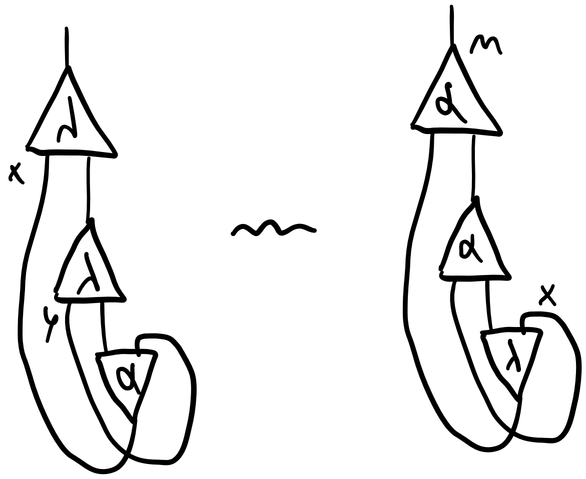
$(\lambda y . y)_{k_1}$

$(k_1\ x)_{k_2}$

$(\lambda x . k_2)_{out}$

In CC hier nur
wice, weil beide ks
positiv sind?!

$(\lambda xy.\, x\, y)\, \square$ $\qquad$ $((\square\, \lambda x.\ \ )\, x)$



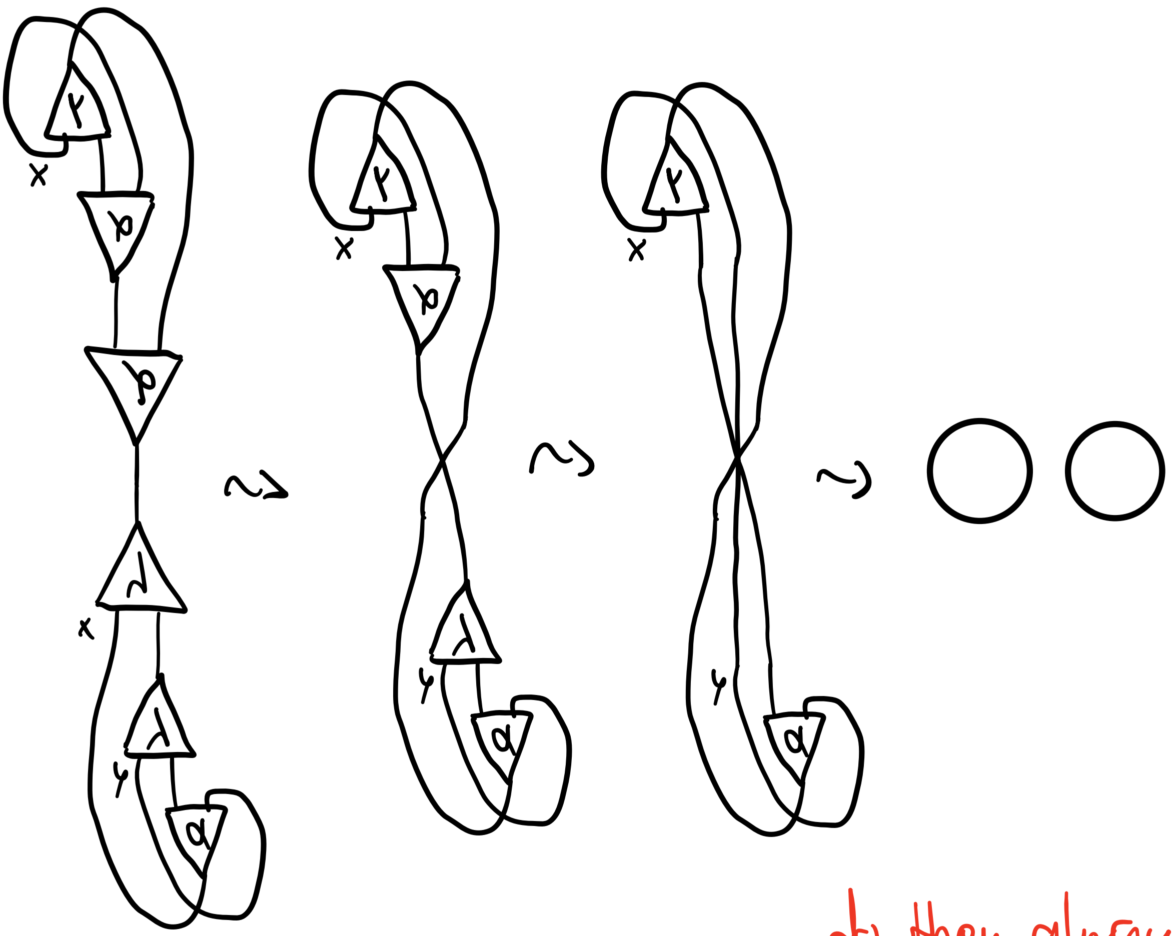
was braucht man in LC, um solche Terme sinnvoll zu beschreiben?

1. flip immer noch well-typed
2. LC intepretation nicht mehr zwingend existent

~ bringt das irgendwas?

was ist das Set an Termen mit einem validen Inversen?

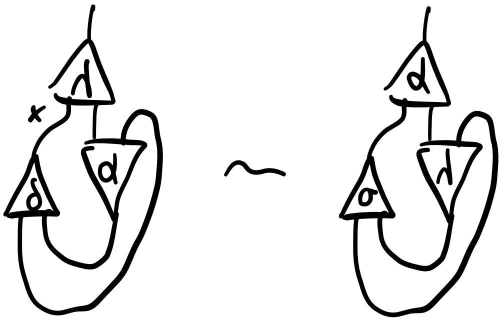
was passiert wenn man sie miteinader verbindet?

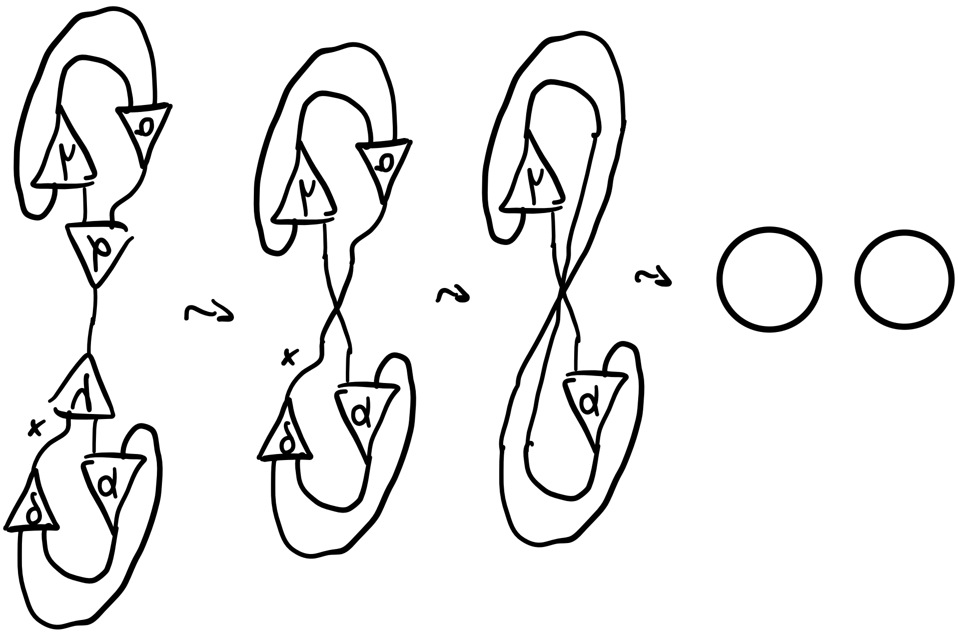


do they always annihilate?

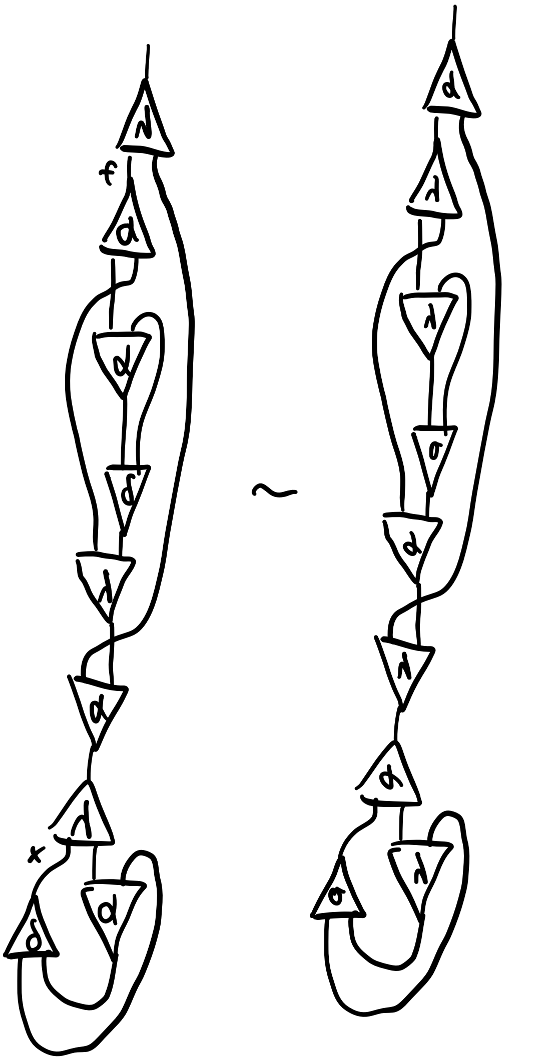
what does the number of circles mean?

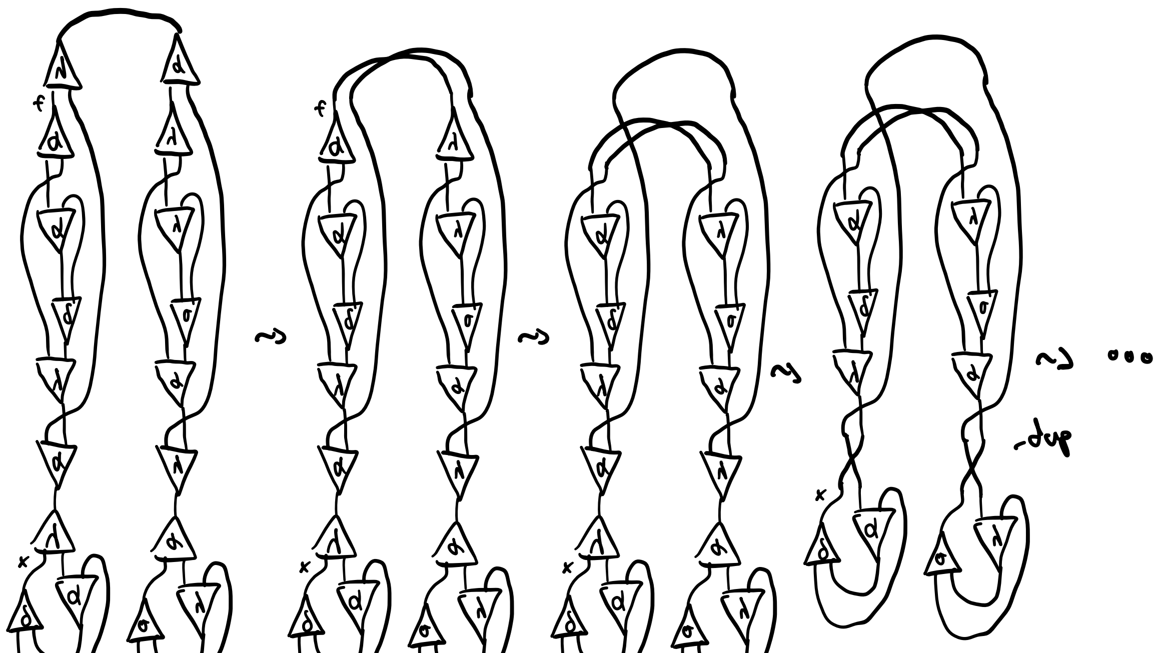
$\lambda x.(x\ x)$ (□





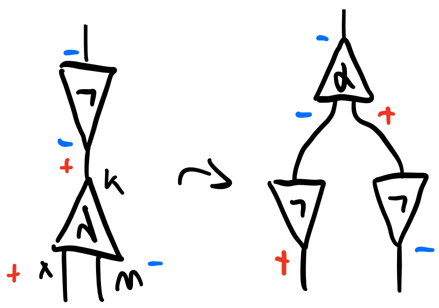
$\lambda f.(\lambda x.(x\ x)\ \ (\lambda x.\ f(x\ x)))$



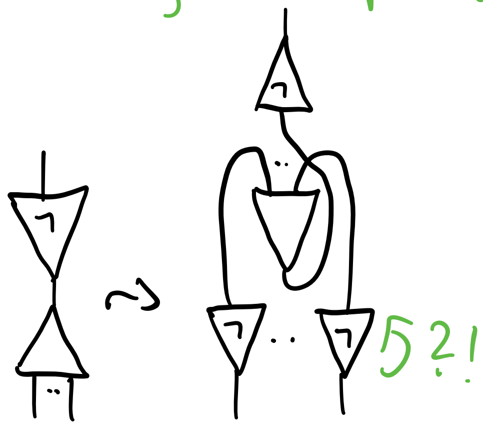


Could we use this for better readback from Lambdascope etc.?
e.g. via explicit ¬(..) operator that flips polarities
Motivation is $\sigma <-> \delta$

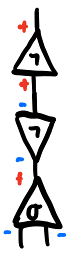
Negation operator?



invariant idea:
all co-agents are upside down

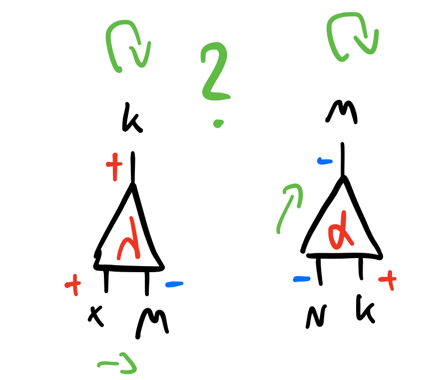


insert before every σ

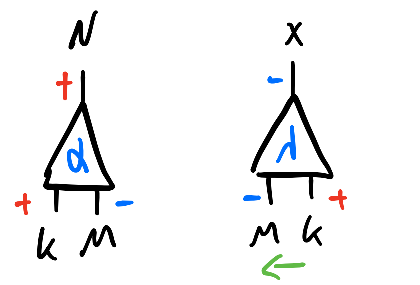


then reduce one step everywhere

Choudhury, Gay



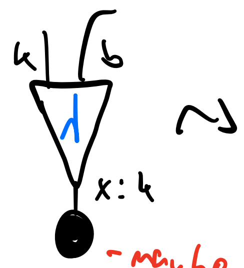
(fn x => M) (M N)



(M @ k) (cofn k => M)

N x

(cofn k => b) ⇝ INR b



⇝ ?!

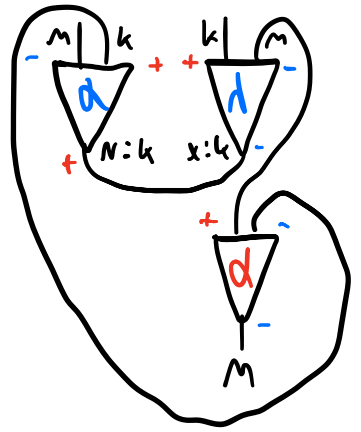
- maybe don't erase k?

rotate?

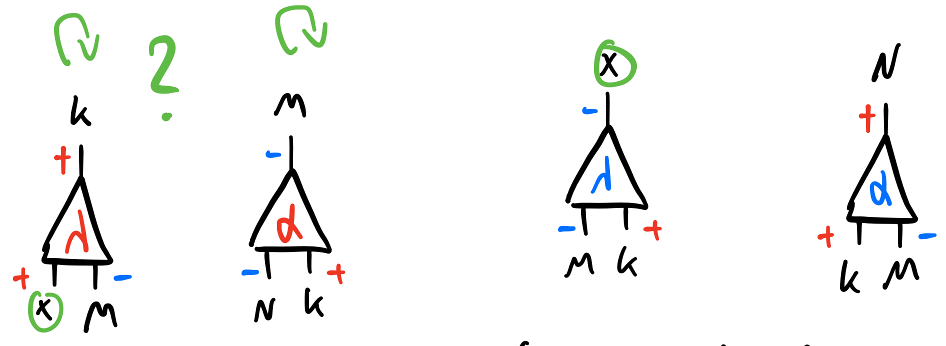
trd : (cofn k => k) ~ choice between value and covalue?!

cofn k => ··M·· ((INL 0) @ k) ⇝ INL 0

INL 0?



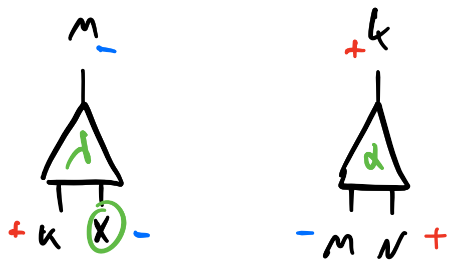
trinity



(fn x => M) (M N) (cofn k => M) (M -> k)

x N

2!



(cofn t => M) (k ⇐ M)

X N

λ_1, α_2 α_3

+ + -

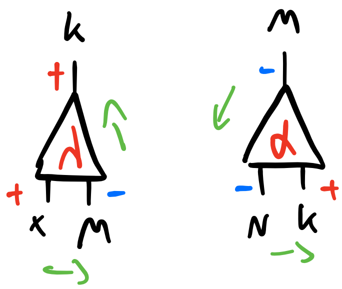
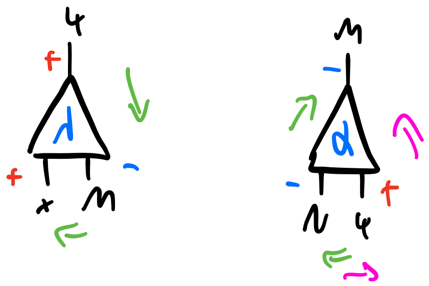
+ - - x x x

α_1, λ_2 λ_3

- - +

- + + - - -

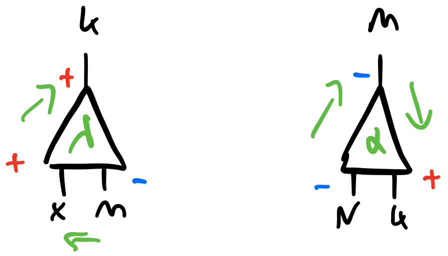
purely rotative trinity + flow "focus views"

$(\text{fn } x \Rightarrow M)_k$ $(M\ N)_k$ $(\text{cofn } k \Rightarrow M)_x$ $(k\ N)_M$

$N\ k$

?



$(\text{fn } M \Rightarrow x)_k$ $(N\ M)_k$

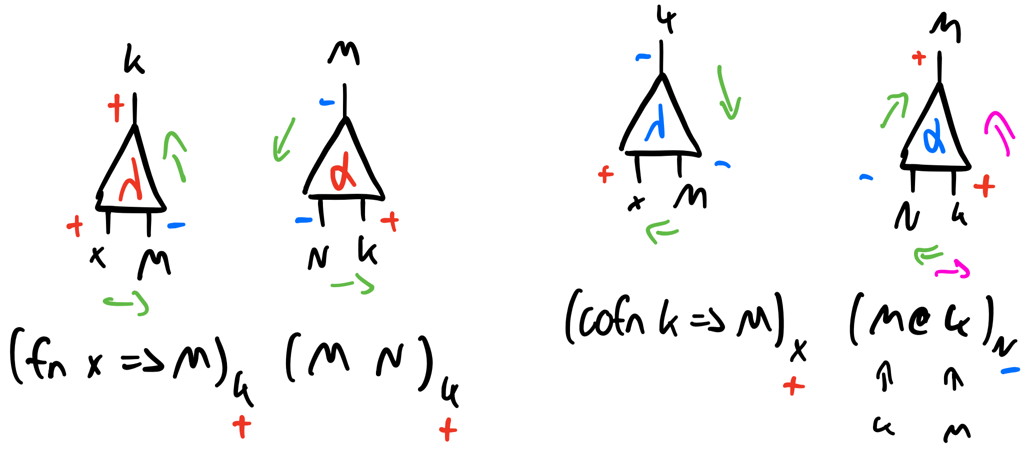
only useful for describing
the normal lc / lnet

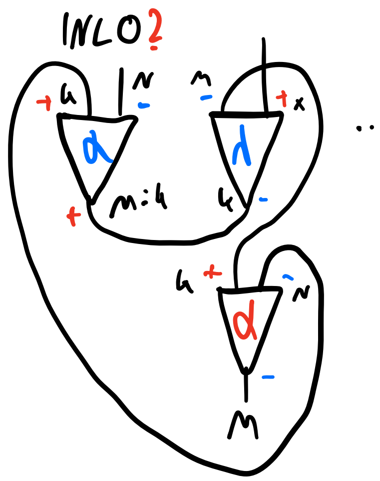
$(x \Rightarrow M)_k$

$(M \Rightarrow k)_x$

$(k \Rightarrow x)_M$

polarity only





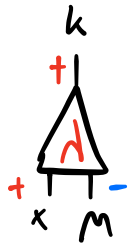
cofn k => ··M·· ((INL 0) @ k)

cont. in <u>rotating agents</u>

B

1.	$(x_+ \Rightarrow m_-)_{k_+}$	$(m_- \; n_-)_{k_+}$	a.
~~2.~~	$(k_+ \Rightarrow m_-)_{x_+}$	$(k_+ \; n_-)_{m_-}$	~~b.~~
~~3.~~	$(m_- \Rightarrow x_+)_{k_+}$	$(k_+ \; m_-)_{n_-}$	c.
4.	$(k_+ \Rightarrow x_+)_{m_-}$	$(n_- \; m_-)_{k_+}$	~~d.~~
~~5.~~	$(x_+ \Rightarrow k_+)_{m_-}$	$(m_- \; k_+)_{n_-}$	~~e.~~
6.	$(m_- \Rightarrow k_+)_{x_+}$	$(n_- \; k_+)_{m_-}$	f.

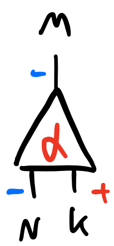
		k	x	k	m	m	x
		1	~~2~~	~~3~~	4	~~5~~	6
k	a	✓	✗	✓	✗	✗	✗
m	~~b~~	✗	✓	✗	✓	✓	✓
n	c	✓	✗	✓	✗	✗	✗
k	~~d~~	✓	✗	✓	✗	✗	✗
n	~~e~~	✓	✗	✓	✗	✗	✗
m	f	✗	✓	✗	✓	✓	✓



$(\lambda x_+ . m_-)_{k_+}$

$(\lambda m_- . k_+)_{x_+}$

$(\lambda k_+ . x_+)_{m_-}$



$(m_- \; n_-)_{k_+}$

$(n_- \; k_+)_{m_-}$

$(k_+ \; m_-)_{n_-}$

Ω = out k

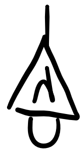
↳ not enough

zero would be nameless at all, really?

but names are just used for wires

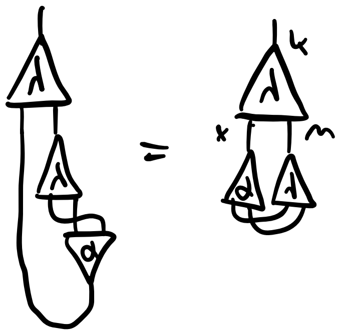
⇒ so always used exactly twice?

$\lambda x . x$



$(\lambda x . x)_k$

$\lambda x . \lambda y . x\ y$



$(\lambda x . m)_k$

$(N\ k)_m \quad (\lambda x . m)_k$

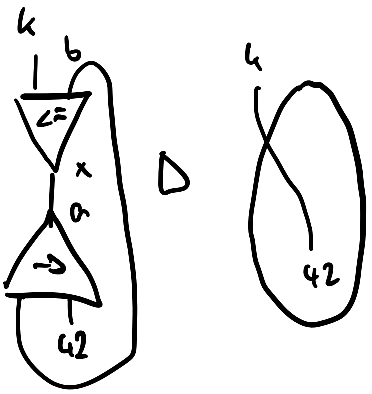
$\Rightarrow (\lambda (x\ y) . (\lambda x . y))_k$

$((\lambda \overset{x}{\_} . \overset{m}{a})_k\ (\lambda \overset{n}{a} . \overset{k}{out})_x)$

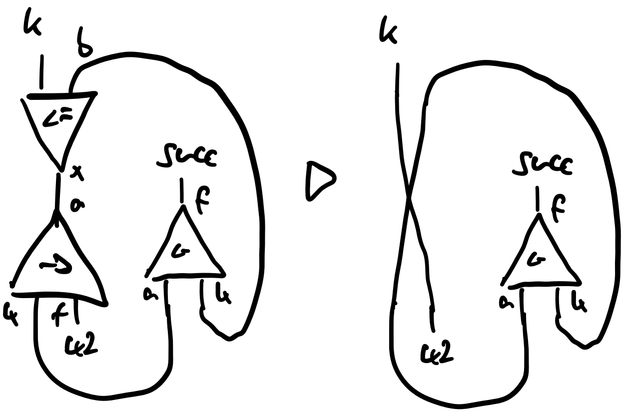
↳ can this be uniquely decoded?

1. Resume once

$x \Leftarrow 42 \to x$

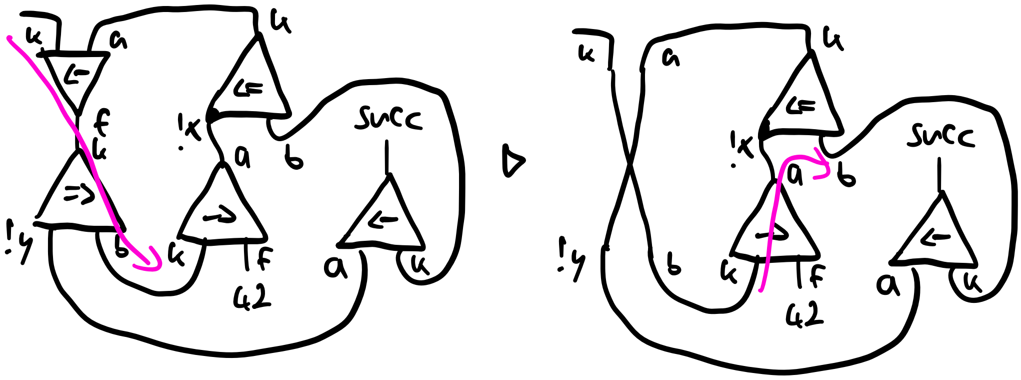


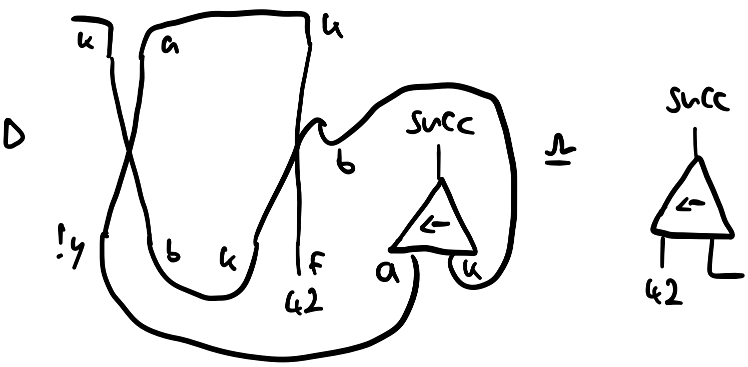
$x \Leftarrow succ \leftarrow (42 \to x)$



$(!y \Rightarrow (42 \to !x) \leftarrow !x \Leftarrow (succ \leftarrow !y))$

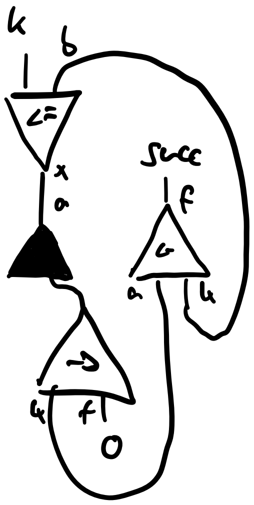
▷ 43

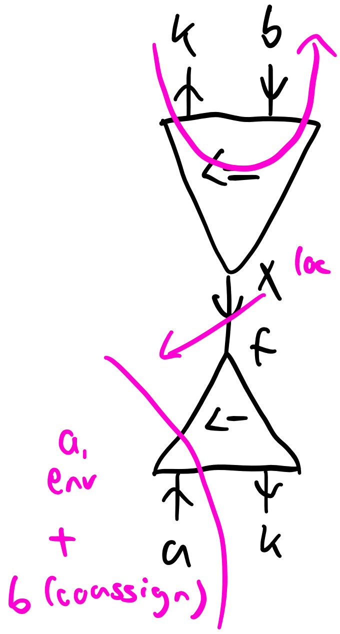


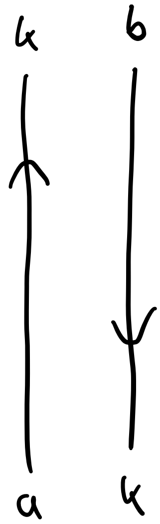


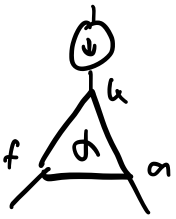
!k <= (succ <- (0 -> !k))

!k <- 10







observation: in  left means CbN, right CbV

1. What with k?
2. Trinity relation to CbV/CbN duality

2 Wadler

$M, N ::= x \mid x \Rightarrow N \mid (S).\alpha$

$K, L ::= \alpha \mid M \leftarrow L \mid x.(S)$

$S, T ::= M \cdot K$

duality

$(x)^\circ \equiv x^\circ$

$((S).\alpha)^\circ \equiv \alpha^\circ.(S^\circ)$

$(\alpha)^\circ \equiv \alpha^\circ$

$(x.(S))^\circ \equiv (S^\circ).x^\circ$

$(M \cdot K)^\circ \equiv M^\circ \cdot K^\circ$

$M^{\circ\circ} \equiv M, \quad K^{\circ\circ} \equiv K, \quad S^{\circ\circ} \equiv S$

CbV

Values $V, W ::= x \mid x \Rightarrow N$

$x \Rightarrow N \cdot V \leftarrow L \xrightarrow{v} V \cdot x.(N \cdot L)$

$V \cdot x.(S) \xrightarrow{v} S[V/x]$

$(S).\alpha \cdot K \xrightarrow{v} S[K/\alpha]$

CbN

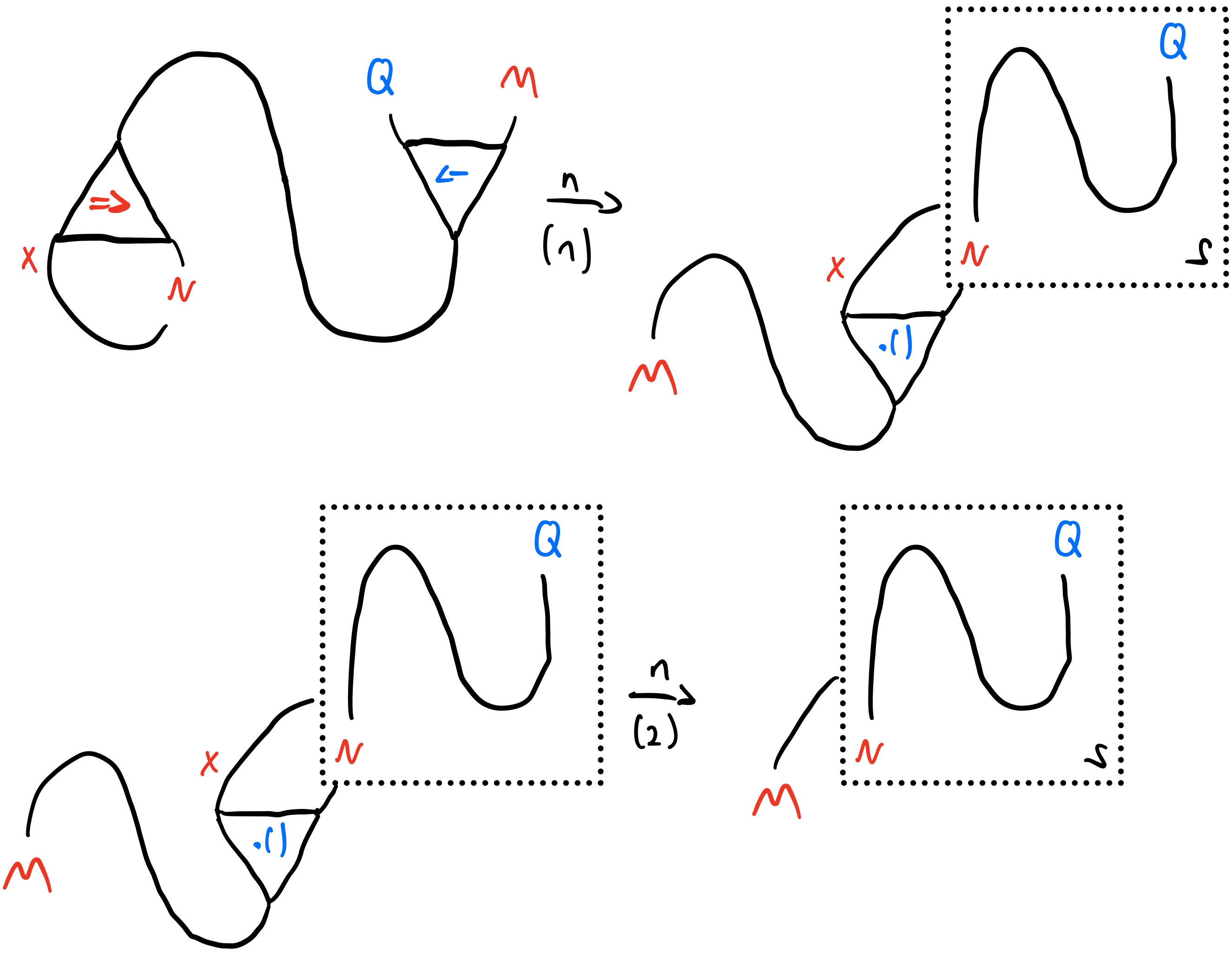
Covalues $P, Q ::= \alpha \mid M \leftarrow Q$

$x \Rightarrow N \cdot M \leftarrow Q \xrightarrow{n} M \cdot x.(N \cdot Q)$

$M \cdot x.(S) \xrightarrow{n} S[M/x]$

$(S).\alpha \cdot P \xrightarrow{n} S[P/\alpha]$

$M \Leftarrow L$ consumes a function $A \to B$ by passing it $M : A$ as argument and then passing the result to be consumed by $L : B$



$(\lambda x.\, x\ x)\ \lambda y. y$

$((x \Rightarrow (x \bullet (x \Leftarrow \alpha)).\alpha) \bullet ((y \Rightarrow y) \Leftarrow \gamma)).\gamma$

$\rhd ((y \Rightarrow y) \bullet x.((x \bullet (x \Leftarrow \alpha)).\alpha) \bullet \gamma)).\gamma$

$\rhd ((x \bullet (x \Leftarrow \alpha)).\alpha) \bullet \gamma).\gamma\ [(y \Rightarrow y)/x]$

$\rhd (((y \Rightarrow y) \bullet ((y \Rightarrow y) \Leftarrow \alpha)).\alpha) \bullet \gamma).\gamma$

$\rhd ((y \Rightarrow y) \bullet ((y \Rightarrow y) \Leftarrow \alpha)).\gamma\ [\gamma/\alpha]$

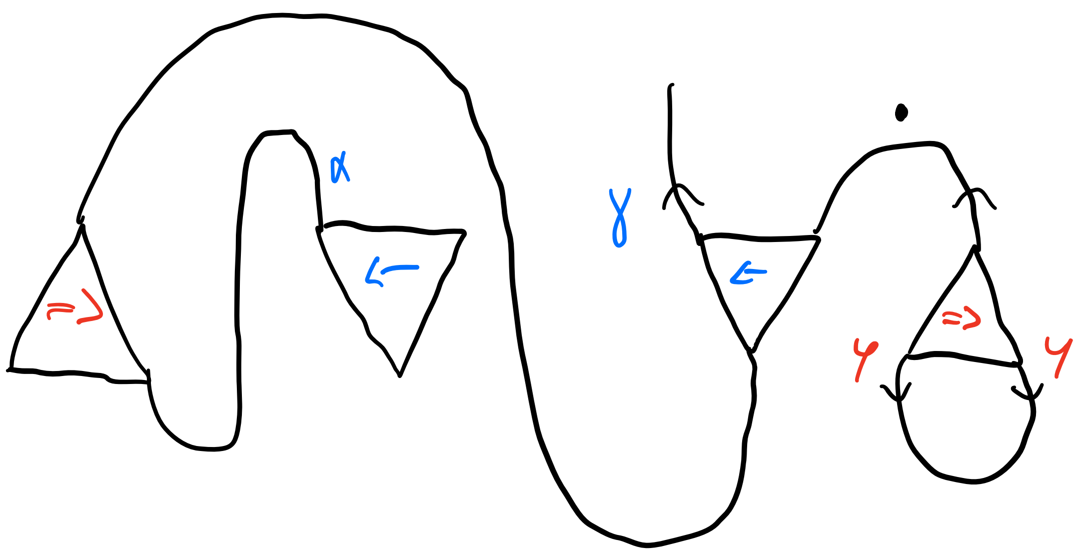
$\rhd ((y \Rightarrow y) \bullet ((y \Rightarrow y) \Leftarrow \gamma)).\gamma$

$\rhd ((y \Rightarrow y) \bullet (y.(y \bullet \gamma))).\gamma$

$\rhd ((y \bullet \gamma)).\gamma\ [(y \Rightarrow y)/y]$

$\rhd ((y \Rightarrow y) \bullet \gamma).\gamma$

$f \bullet a \Leftarrow \gamma$



Paper

FP/ND: intro/elim of producers

SC: right elimination → left introduction (consumers)

LC: one producer, one consumer
  ↳ multiple, CPS/callcc

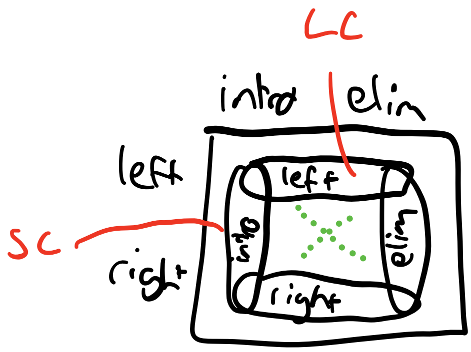
intro vs. elim
(both left)
  ↳ both "producing"

SC: producing and consuming terms
  elimination rule → left rules on consumers

  $\mathrm{Fst} \,;\, f : \langle (e_1, e_2) \,\|\, \mathrm{Fst} \,;\, f \rangle \triangleright \langle e_1 \,\|\, f \rangle$

  ↳ inside-out structure

left vs. right
(both intro)



$$\begin{array}{lll} & \overset{\text{elim}}{\downarrow} & \overset{\text{intro}}{\downarrow} \\ \text{Left:} & \langle x \,\|\, \mathrm{Right} \,;\, \alpha \rangle, & \langle z \,\|\, \mathrm{Match} \cdots \rangle \\ \text{Right:} & \langle \mathrm{Right}\ x \,\|\, \alpha \rangle, & \mathrm{Case}\ z \cdots \\ & \underset{\text{intro}}{\uparrow} & \underset{\text{elim}}{\uparrow} \end{array}$$

biexpressibility:
left intro = right elim
right intro = left elim
  ↳ "as powerful"

Calculus	Program	
Right	Case z {Left $x \mapsto \langle$Right $x \,\|\, \alpha\rangle$; Right $y \mapsto \langle$Left $y \,\|\, \alpha\rangle$}	intro + elim (right)
Intro	$\langle z \,\|$ Match {Left $x \mapsto \langle$Right $x \,\|\, \alpha\rangle$; Right $y \mapsto \langle$Left $y \,\|\, \alpha\rangle\rangle$	intro (left + right)
Left	$\langle z \,\|$ Match {Left $x \mapsto \langle x \,\|$ Right $\fatsemi\, \alpha\rangle$; Right $y \mapsto \langle y \,\|$ Left $\fatsemi\, \alpha\rangle$}$\rangle$	intro + elim (left)
Elim	Case z {Left $x \mapsto \langle x \,\|$ Right $\fatsemi\, \alpha\rangle$; Right $y \mapsto \langle y \,\|$ Left $\fatsemi\, \alpha\rangle$}	elim (left + right)

⤙ <-> →

& <-> ⊕

⅋ <-> ⊗

dual

units

& -> ⊤

⅋ -> ⊥

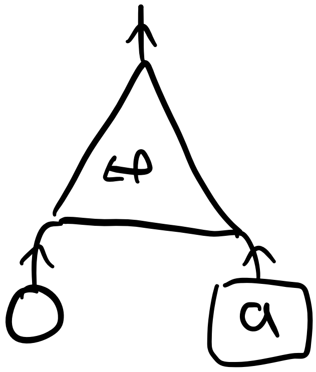
⊗ -> 1

⊕ -> 0

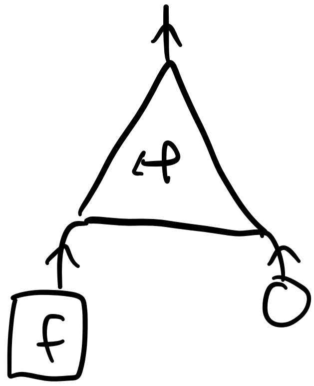
dual

Inr / Inl

Inr a

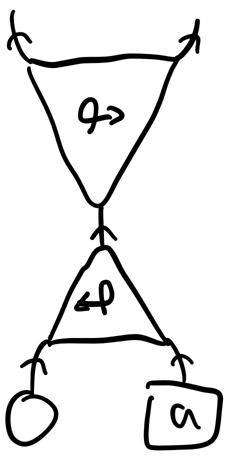


Inl f



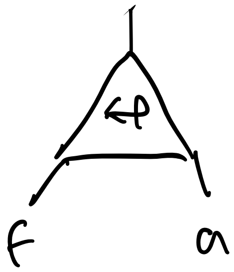
Sum

$k \Leftarrow ((\text{match } \{ hf \to k \ ; \ ha \to k \}) \leftarrow \text{Inr } a)$



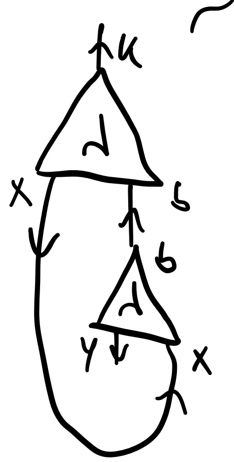
$\boxed{hf}$

product?



$K \triangleq \lambda x\, y\, .\, x$

prompt



$k[x\ [y\ x]]$; $k[[b\ y]\ b]$

$y[x\ [k\ x]]$; $y[[b\ k]\ b]$

$x[[k\ x]\ y]$; —

$x[[y\ x]\ k]$; —

$b[k\ [b\ y]]$; —

$b[y\ [b\ k]]$; —

flipped wire pols!

(x^+, b^-, k^+)

(f^-, a^-, k^+)

		view			view
+:	$[x^+\ b^-]$	k	−:	$(a^-\ k^+)$	f
	$[b^-\ k^+]$	x		$(k^+\ f^-)$	a
	$(f^-\ a^-)$	k		$[k^+\ x^+]$	b

m^- : term *is* negative, color indicates counterpart polarity

$$\frac{\Gamma \vdash e_1 : - \qquad \Gamma \vdash e_2 : -}{\Gamma \vdash [e_1\ e_2] : -}$$

$$\frac{\Gamma \vdash e_1 : - \qquad \Gamma \vdash e_2 : +}{\Gamma \vdash [e_1\ e_2] : +} \qquad \frac{\Gamma \vdash e_1 : - \qquad \Gamma \vdash e_2 : +}{\Gamma \vdash (e_1\ e_2) : -}$$

$$\frac{\Gamma \vdash e_1 : + \qquad \Gamma \vdash e_2 : -}{\Gamma \vdash [e_1\ e_2] : +} \qquad \frac{\Gamma \vdash e_1 : + \qquad \Gamma \vdash e_2 : -}{\Gamma \vdash (e_1\ e_2) : -}$$

$$\frac{\Gamma \vdash e_1 : + \qquad \Gamma \vdash e_2 : +}{\Gamma \vdash (e_1\ e_2) : +}$$

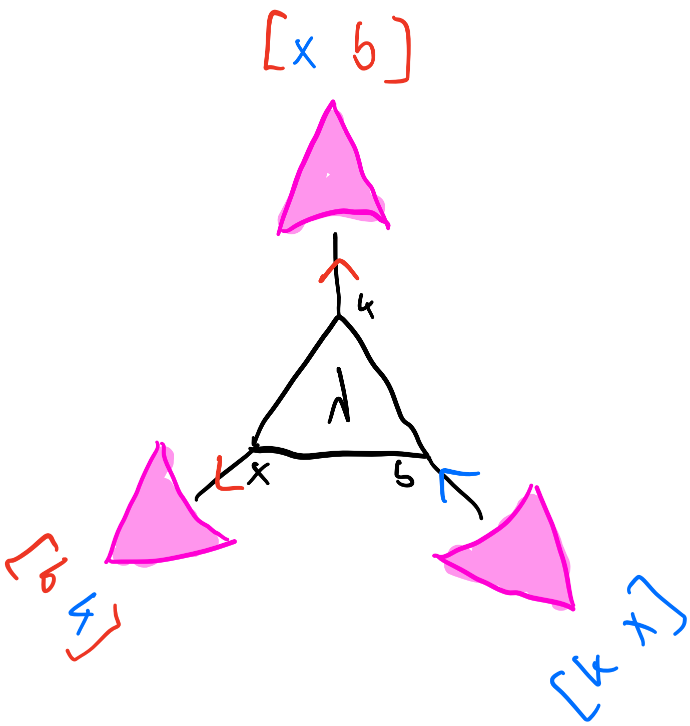
$$\frac{}{\Gamma \vdash k : -}$$

$$\frac{\Gamma \vdash x : - \qquad \Gamma \vdash e_2 : +}{\Gamma \vdash x\ e_2 : +}$$

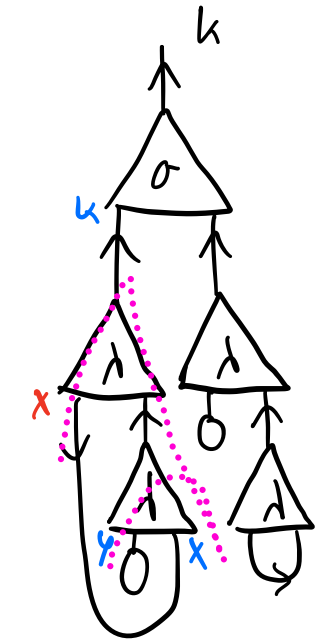
$$k\,[x\,[y\;x]]$$

$$\dfrac{\dfrac{}{\vdash k\ -} \qquad \dfrac{\dfrac{}{x \vdash x\ -} \qquad \dfrac{\dfrac{}{y \vdash y\ -} \quad \dfrac{}{x \vdash x\ +}}{y, x \vdash [y\;x]\ +}\ \checkmark}{y \vdash [x\,[y\;x]]\ +}\ \checkmark}{y \vdash k[x\,[y\;x]]\ +}\ \checkmark$$

$$\dfrac{y \vdash y : - \qquad \dfrac{x \vdash x : + \qquad \dfrac{\vdash k : - \quad x \vdash x : -}{x \vdash [k\;x] : -}\ \checkmark}{\vdash [x\,[k\;x]] : +}\ \checkmark}{y \vdash y\,[x\,[k\;x]] : +}\ \checkmark$$



```
def sup(f) {
    f(0) + f(1)
}
```



views!

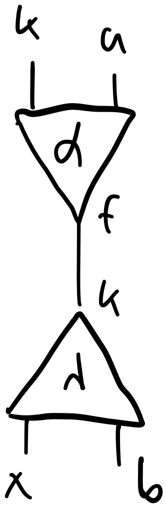
$$\overset{-}{y}[\overset{+}{x}[\overset{-}{k}\ \overset{-}{x}]]$$

$$\overset{-}{y}[\overset{+}{x}[\underbrace{\overset{-}{k}[\overset{-}{a}[\overset{-}{b}\ \overset{+}{b}]]}_{-\ \text{but}\ +!}\ \overset{-}{x}]]$$

$$\Rightarrow\ \overset{-}{y}[\overset{+}{x}[\underbrace{\neg\overset{-}{k}[\overset{-}{a}[\overset{-}{b}\ \overset{+}{b}]]}\ \overset{-}{x}]]$$

$k \Leftrightarrow f$

Redexes



$[x\ b]_k \bowtie (f\ a)_k$: $([x\ b]\ a)$

$[x\ b]_k \bowtie (a\ k)_f$: /

$[x\ b]_k \bowtie (k\ f)_a$: $(k\ [x\ b])$

$[b\ k]_x \bowtie (f\ a)_k$: /

$[b\ k]_x \bowtie (a\ k)_f$: $[b\ (a\ k)]$

$[b\ k]_x \bowtie (k\ f)_a$: /

$[k\ x]_b \bowtie (f\ a)_k$: /

$[k\ x]_b \bowtie (a\ k)_f$: $[(a\ k)\ x]$

$[k\ x]_b \bowtie (k\ f)_a$: /

	$[k\ x]$	$[x\ b]$	$[b\ k]$
$(f\ a)$	✗	✓	✗
$(a\ k)$	✓	✗	✓
$(k\ f)$	✗	✓	✗

$$([x\ b]\ a)_k \rhd b, \quad x \mapsto a$$

$$(k\ [x\ b])_a \rhd x, \quad k \mapsto b$$

$$[b\ (a\ k)]_x \rhd a, \quad k \mapsto b$$

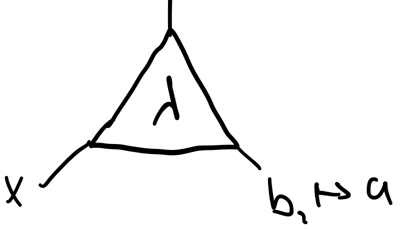
$$[(a\ k)\ x]_b \rhd k, \quad x \mapsto a$$

$$([x_1\ b_1]\ [x_2\ b_2]) \rhd b_1, \quad x_1 \mapsto [x_2\ b_2]$$

$$([[k\ x]\ b]\ a) \rhd b, \quad [k\ x]_{b_2} \mapsto a$$

$$k \mapsto [x\ a]$$

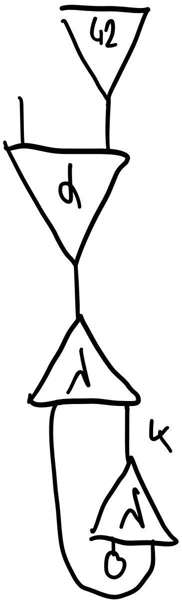
$$x \mapsto [a\ k]$$

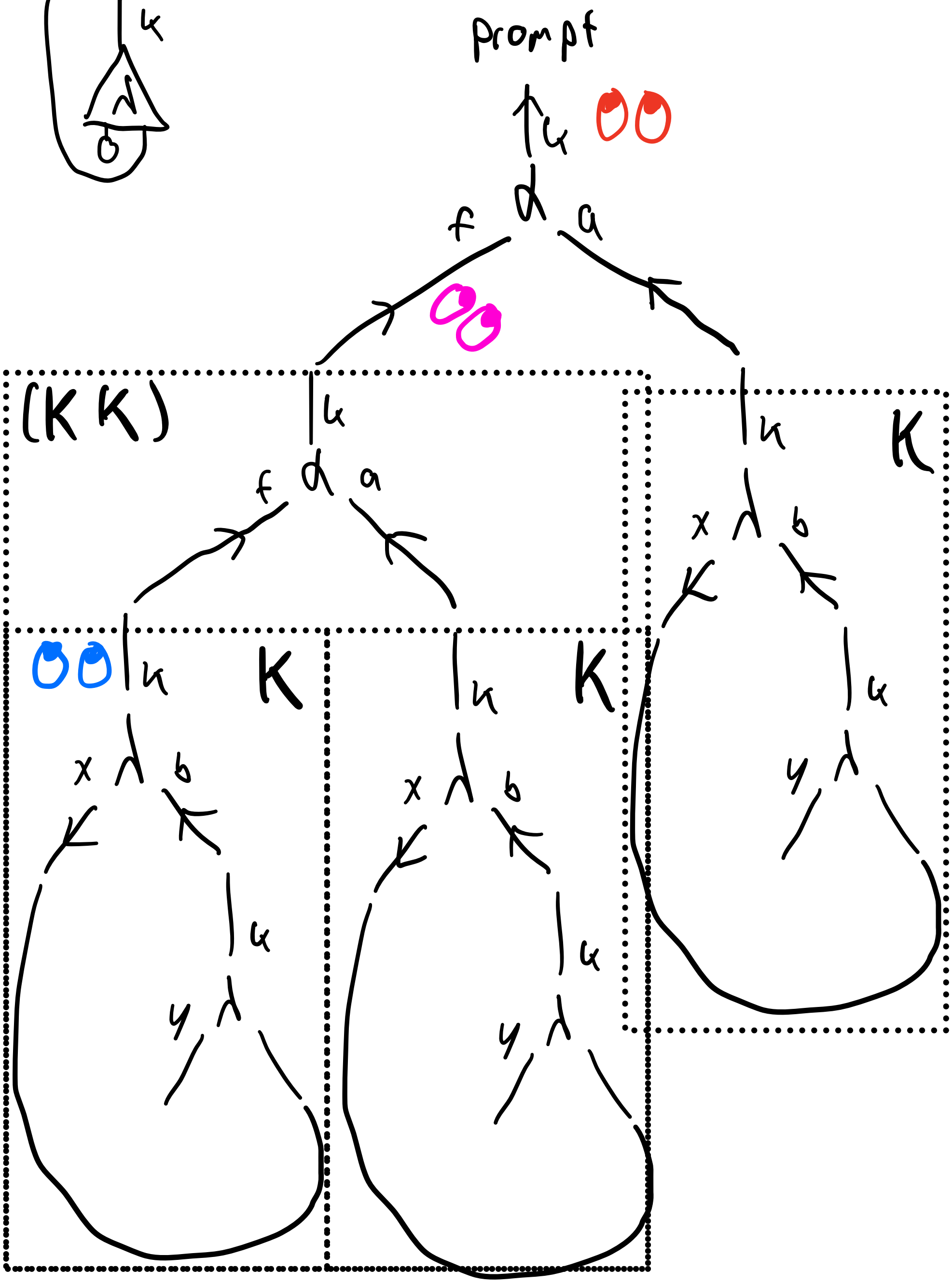


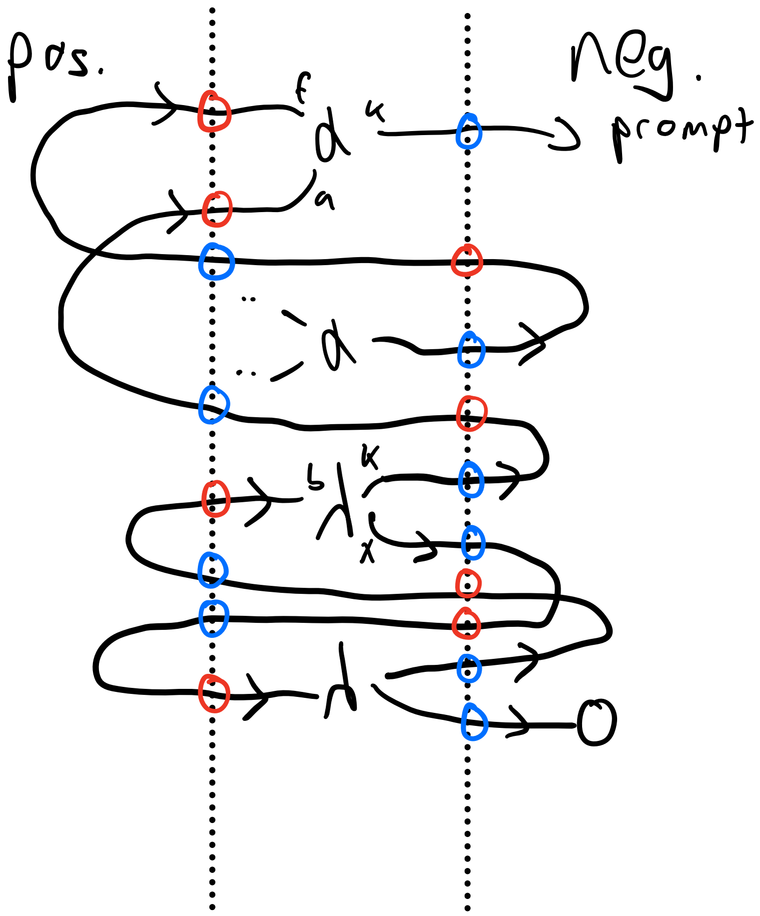
$$([[k\ x]\ k^c]\ 42) \triangleright k^c_1 \qquad [k\ x] \mapsto 42$$

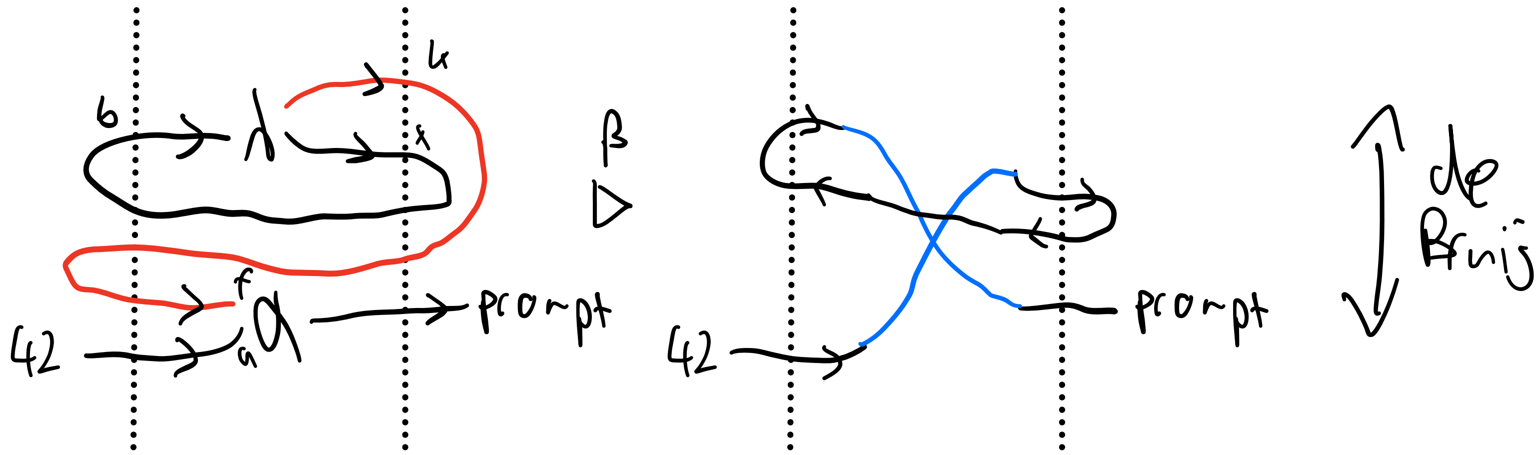
$$\rightsquigarrow [x\ 42] \qquad \Rightarrow x^c \mapsto [42\ k]$$

$$k^c \mapsto [x\ 42]$$









⇒ slot machine